JOURNAL OF COMPUTATIONAL PHYSICS **141,** 37–45 (1998)
ARTICLE NO. CP985905

A Particle Algorithm for Linear Kinetic Analysis in Tokamak Plasmas

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Received May 5, 1997

A particle algorithm for linear kinetic analysis in tokamak plasmas is developed. Linear kinetic stability of a tokamak plasma is analyzed as an initial value problem. Particles are used to sample plasma elements along equilibrium characteristics in 4-dimensional phase space (R, z, $v_{\parallel}$, μ). Each particle is accompanied with a weight which is a function of the toroidal angle. Integrals in the phase space are evaluated through the weight function and the particle location in the 4-dimensional space. Destabilization of an $n = 2$ toroidal Alfvén eigenmode is investigated as a test of the algorithm, and convergence in number of used particles is assessed.

Key Words: kinetic stability; particle algorithm; tokamak plasma.

1. INTRODUCTION

Kinetic analysis on magnetically confined plasmas is not so easy since the phase space concerned has a high dimension which demands large computational resources to analyze. Another difficulty is the complicated characteristics of the Vlasov equation. The characteristics are nothing but the particle orbits. For example, particle orbits deviate from magnetic surfaces, and passing and trapped particles show qualitatively different behaviors from each other. Thus, even the linear analysis is not a simple task.

Turning to nonlinear analysis, particle simulation has been a powerful tool for the investigation of nonlinear kinetic phenomena in fusion and space plasmas. Super-particles are used in particle simulations, and the number of super-particles is much less than the number of particles in real plasmas. Since the electromagnetic field is coupled with low-order velocity moments in the phase space, charge density, and current density, the required number of super-particles can be reasonable. The small number of super-particles, however, lead to a thermal fluctuation much larger than that in a real plasma. Large thermal fluctuation, called numerical noise, makes it difficult to deal with a phenomenon with small amplitude.

0021-9991/98 $25.00

Recently, this drawback has been relaxed by the so-called δf algorithm [1–3], which was developed in the fusion community. Furthermore, Aydemir [4] gave a clear interpretation of the δf algorithm from a viewpoint of Monte Carlo simulation.

The δf algorithm gives a fine prospect also of linear properties. The linear property of an instability can be studied with the δf particle simulation as an initial value problem if initial perturbations are set to be sufficiently small.

The δf algorithm, however, can be improved for linear analysis. In this paper, we develop a particle simulation algorithm for linear kinetic phenomena in tokamak plasmas. With this algorithm computational resources required in the δf particle simulation can be reduced. Super-particles are used as markers in the phase space. These marker particles are followed along their equilibrium orbits, and each of them is accompanied with a weight function. Integrals in the phase space such as number density, current density, and pressures are evaluated through the particle location and the weight function. Although this new algorithm is similar to the δf particle algorithm, an essential difference is that particle weights are functions of independent variables on which the equilibrium distribution is symmetric.

For example, in an axisymmetric plasma such as a tokamak plasma, the weight is a function of the toroidal angle. Toroidal location of each particle has no meaning. Generally, the phase space of a tokamak plasma with the drift-kinetic or the gyro-kinetic description is a five-dimensional space $(R, \varphi, z, v_\parallel, \mu)$ which demands large computational resources to study kinetic phenomena. The new algorithm reduces the phase space for particles to four-dimensions $(R, z, v_\parallel, \mu)$. Thus the required number of particles also can be reduced.

This new simulation algorithm can be applied to general linear kinetic phenomena of plasmas which have a symmetric direction(s) in equilibrium. In this paper, we apply it to the toroidal Alfvén eigenmode (TAE mode) [5] in a tokamak plasma. The TAE mode has attracted many scientists from the viewpoints of the basic plasma physics on the wave-particle interaction and the confinement of energetic alpha particles in fusion devices.

We adopt a hybrid model in which plasma is divided into two parts, the energetic particles and the background plasma. The background plasma is described as a magnetohydrodynamic (MHD) fluid. We couple the linear particle algorithm with a linearized MHD code employing this hybrid model. Linear simulations are carried out for an $n = 2$ TAE mode. The mode structure, real frequency, and linear growth rate are obtained.

The linear particle algorithm is described in Section 2. Section 3 is devoted to the plasma model and the results of the linear simulation for the $n = 2$ TAE mode. Summary and discussion are given in Section 4.

2. A LINEAR PARTICLE ALGORITHM

We employ the guiding-center approximation. Let us start by describing the guiding-center velocity $\mathbf{u}$ for a particle in an electromagnetic field $\mathbf{E}$ and $\mathbf{B}$,

$$\mathbf{u} = \mathbf{v}_\parallel^* + \mathbf{v}_E + \mathbf{v}_B, \tag{1}$$

$$\mathbf{v}_\parallel^* = v_\parallel[\mathbf{b} + \rho_\parallel \boldsymbol{\nabla} \times \mathbf{b}], \tag{2}$$

$$\mathbf{v}_E = \frac{1}{B}[\mathbf{E} \times \mathbf{b}], \tag{3}$$

$$\mathbf{v}_B = \frac{1}{qB}[-\mu \boldsymbol{\nabla} B \times \mathbf{b}], \tag{4}$$

$$\rho_\parallel = \frac{mv_\parallel}{qB}, \tag{5}$$

$$\mathbf{b} = \frac{\mathbf{B}}{B}, \tag{6}$$

$$mv_\parallel \frac{dv_\parallel}{dt} = \mathbf{v}_\parallel^* \cdot [q\mathbf{E} - \mu \nabla B], \tag{7}$$

where m, q, $v_\parallel$, and μ are the particle mass, particle charge, parallel velocity, and magnetic moment, respectively.

From Eqs. (1)–(7) we obtain the drift kinetic equation which describes the time evolution of the distribution function in the phase space $(R, \varphi, z, v_\parallel, \mu)$,

$$\frac{\partial}{\partial t} f + \frac{1}{B}\nabla \cdot (B\mathbf{u} f) + \frac{\partial}{\partial v_\parallel}\left(\frac{dv_\parallel}{dt} f\right) + f \frac{\partial}{\partial t} \ln B = 0. \tag{8}$$

We neglect for simplicity the compressibility in the phase space with the present guiding-center approximation. Namely, we assume that the following relation is always satisfied:

$$\frac{1}{B}\nabla \cdot (B\mathbf{u}) + \frac{\partial}{\partial v_\parallel}\left(\frac{dv_\parallel}{dt}\right) + \frac{\partial}{\partial t} \ln B = 0. \tag{9}$$

Equation (8) can be simplified with the aid of this assumption into,

$$\frac{\partial}{\partial t} f + \mathbf{u} \cdot \nabla f + \frac{dv_\parallel}{dt}\frac{\partial}{\partial v_\parallel} f = 0. \tag{10}$$

For linear analysis the distribution function f is divided into two parts, the initial equilibrium distribution f_0 and the deviation δf. Equation (10) is then linearized into

$$\frac{\partial}{\partial t}\delta f + \mathbf{u}_0 \cdot \nabla \delta f + \left(\frac{dv_\parallel}{dt}\right)_0 \frac{\partial}{\partial v_\parallel}\delta f = -\left[\mathbf{u}_1 \cdot \nabla f_0 + \left(\frac{dv_\parallel}{dt}\right)_1 \frac{\partial}{\partial v_\parallel} f_0\right], \tag{11}$$

$$\mathbf{u}_0 = v_\parallel[\mathbf{b}_0 + \rho_\parallel \nabla \times \mathbf{b}_0] + \frac{1}{qB}[-\mu \nabla B \times \mathbf{b}_0], \tag{12}$$

$$\left(\frac{dv_\parallel}{dt}\right)_0 = [\mathbf{b}_0 + \rho_\parallel \nabla \times \mathbf{b}_0] \cdot \left[-\frac{\mu}{m}\nabla B\right], \tag{13}$$

$$\mathbf{u}_1 = v_\parallel \delta\mathbf{b} + \frac{1}{B}[\mathbf{E} \times \mathbf{b}_0], \tag{14}$$

$$\left(\frac{dv_\parallel}{dt}\right)_1 = [\mathbf{b}_0 + \rho_\parallel \nabla \times \mathbf{b}_0] \cdot \left[\frac{q}{m}\mathbf{E}\right] + \delta\mathbf{b} \cdot \left[-\frac{\mu}{m}\nabla B\right], \tag{15}$$

$$\delta\mathbf{b} = \mathbf{b} - \mathbf{b}_0, \tag{16}$$

where we neglect the deviation of magnetic field intensity "δB" since many of the interested modes in tokamak plasmas are incompressible. Furthermore, the term $v_\parallel \rho_\parallel \nabla \times \delta\mathbf{b}$ is neglected in Eq. (14) since it is much smaller than $v_\parallel \delta\mathbf{b}$.

Let us consider an integral of the form

$$\delta I(A) \equiv \int_V A \delta f \mathcal{J} \, dR \, d\varphi \, dz \, dv_\parallel \, d\mu, \tag{17}$$

where A is a general function of the phase-space coordinates, V is the phase space volume, and $\mathcal{J} \equiv BR$ is the Jacobian of the transformation from $(R, \varphi, z, v_\parallel, \mu)$ to an appropriate Cartesian coordinate system.

Following Aydemir [4] we develop a particle algorithm to estimate $\delta I(A)$. We express δf in a form of $\delta f = g(R, \varphi, z, v_\parallel, \mu) f_0(R, z, v_\parallel, \mu)$. Equation (17) is transformed into

$$\delta I(A) = \int_0^{2\pi} d\varphi \int_{V_4} Ag f_0 \mathcal{J} dR\, dz\, dv_\parallel\, d\mu, \tag{18}$$

where V_4 is the four-dimensional phase space volume.

According to Eq. (17) of Aydemir [4] a particle-estimate for the second integral of the right-hand side of Eq. (18) is given by

$$\begin{aligned} &\int_{V_4} Ag f_0 \mathcal{J} dR\, dz\, dv_\parallel\, d\mu \\ &\quad \simeq \frac{1}{N} \sum_{j=1}^{N} \frac{A(R_j, \varphi, z_j, v_{\parallel j}, \mu_j) g(R_j, \varphi, z_j, v_{\parallel j}, \mu_j) f_0(R_j, z_j, v_{\parallel j}, \mu_j)}{p(R_j, z_j, v_{\parallel j}, \mu_j)}, \end{aligned} \tag{19}$$

where N is the number of super-particles and p is a probability density in the four-dimensional phase-space volume such that

$$\int_{V_4} p(R, z, v_\parallel, \mu) \mathcal{J} dR\, dz\, d\mu\, dv_\parallel = 1. \tag{20}$$

We adopt the importance sampling method in which the probability density is given by

$$p(R, z, v_\parallel, \mu) = \frac{1}{N_s} f_0(R, z, v_\parallel, \mu), \tag{21}$$

where N_s is the number of physical particles per unit toroidal angle. If we initially distribute super-particles to satisfy the relation above, it is satisfied throughout the analysis.

We define a weight function of the jth particle as

$$w_j(\varphi) \equiv \frac{1}{N} g(R_j, \varphi, z_j, v_{\parallel j}, \mu) \frac{f_0(R_j, z_j, v_{\parallel j}, \mu_j)}{p(R_j, z_j, v_{\parallel j}, \mu_j)}. \tag{22}$$

The particle-estimate for $\delta I(A)$ is

$$\delta I(A) = \int_0^{2\pi} \sum_{j=1}^{N} A(R_j, \varphi, z_j, v_{\parallel j}, \mu_j) w_j(\varphi)\, d\varphi. \tag{23}$$

Our goal is to obtain the equation for the time evolution of the weight function. From Eqs. (21) and (22), the weight function can be related to δf as,

$$\begin{aligned} \delta f(R_j, \varphi, z_j, v_{\parallel j}, \mu_j) &= g(R_j, \varphi, z_j, v_{\parallel j}, \mu_j) f_0(R_j, z_j, v_{\parallel j}, \mu_j) \\ &= \frac{N}{N_s} f_0(R_j, z_j, v_{\parallel j}, \mu_j) w_j(\varphi). \end{aligned} \tag{24}$$

It is convenient to normalize $w_j(\varphi)$ to satisfy

$$\delta f(R_j, \varphi, z_j, v_{\parallel j}, \mu_j) = f_0(R_j, z_j, v_{\parallel j}, \mu_j) w_j(\varphi). \tag{25}$$

We substitute Eq. (25) into Eq. (11) and obtain

$$\frac{D}{Dt} w_j(\varphi) + u_{0\varphi} \frac{\partial}{R\partial\varphi} w_j(\varphi) = -\frac{1}{f_0}\left[\mathbf{u}_1 \cdot \nabla f_0 + \left(\frac{dv_\parallel}{dt}\right)_1 \frac{\partial}{\partial v_\parallel} f_0\right], \tag{26}$$

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \mathbf{u}_{0R}\frac{\partial}{\partial R} + \mathbf{u}_{0z}\frac{\partial}{\partial z} + \left(\frac{dv_\parallel}{dt}\right)_0 \frac{\partial}{\partial v_\parallel}. \tag{27}$$

If the weight function and the electromagnetic field have a form of $e^{in\varphi}$ such as

$$w_j(\varphi) = w_j^\dagger e^{in\varphi}, \tag{28}$$

Eq. (26) is transformed into,

$$\frac{D}{Dt} w_j^\dagger = -\frac{inu_{0\varphi}}{R} w_j^\dagger - \frac{1}{f_0}\left[\mathbf{u}_1^\dagger \cdot \nabla f_0 + \left(\frac{dv_\parallel}{dt}\right)_1^\dagger \frac{\partial}{\partial v_\parallel} f_0\right]. \tag{29}$$

Expansion to multiple toroidal mode numbers is straightforward.

Finally we show the particle-estimates of the first-order number density, current density, parallel and perpendicular pressures. They are given by

$$\delta n = \sum_{j=1}^{N} w_j(\varphi) S(R - R_j) S(z - z_j), \tag{30}$$

$$\delta \mathbf{j} = q \sum_{j=1}^{N} w_j(\varphi) \mathbf{u}_j S(R - R_j) S(z - z_j), \tag{31}$$

$$\delta P_\parallel = \sum_{j=1}^{N} w_j(\varphi) m v_{\parallel j}^2 S(R - R_j) S(z - z_j), \tag{32}$$

$$\delta P_\perp = B \sum_{j=1}^{N} w_j(\varphi) \mu_j S(R - R_j) S(z - z_j), \tag{33}$$

where S is the shape factor of a particle.

3. APPLICATION TO TAE MODE

TAE mode [5] has recently become a focus of attention for fusion physicists, since it can be excited resonantly with alpha particles which are produced from deuterium–tritium reactions. The free energy source for TAE modes is the spatial gradient of the alpha particle density.

Evaluation of the linear growth rate is one of the major issues on TAE mode. It is necessary to know precisely the interaction between alpha particles and TAE modes. The interaction is not simple, since drift-orbits of alpha particles deviate from magnetic surfaces and not only passing particles, but also trapped particles, can resonate with TAE modes.

We apply the linear particle algorithm developed in the previous section to energetic particles which interact with a TAE mode. The plasma is divided into two parts, the background plasma and the energetic particles. The background plasma is described by the ideal MHD equations and the electromagnetic field is given by the MHD description. This is a reasonable approximation under the condition that the alpha density is much less than the background plasma density. This model is first proposed by Ref. [6], and conservation of the total energy is proved in Ref. [7].

The MHD equations with the effect of energetic particles are

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}), \tag{34}$$

$$\rho \frac{\partial}{\partial t}\mathbf{v} + \rho \mathbf{v} \cdot \nabla \mathbf{v} = \left(\frac{1}{\mu_0}\nabla \times \mathbf{B} - \mathbf{j}_h'\right) \times \mathbf{B} - \nabla P, \tag{35}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \tag{36}$$

$$\frac{\partial P}{\partial t} = -\nabla \cdot (P\mathbf{v}) - (\gamma - 1)P\nabla \cdot \mathbf{v}, \tag{37}$$

$$\mathbf{j}_h' = \mathbf{j}_{h\parallel} + \frac{1}{B}(P_{h\parallel}\nabla \times \mathbf{b} - P_{h\perp}\nabla \ln B \times \mathbf{b}) - \nabla \times \left(\frac{P_{h\perp}}{B}\mathbf{b}\right), \tag{38}$$

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B}, \tag{39}$$

where μ_0 is the vacuum magnetic permeability and γ is the adiabatic constant, and all other quantities are conventional.

All MHD variables are linearized around their equilibrium distribution:

$$\frac{\partial \rho_1}{\partial t} = -\nabla \cdot (\rho_0 \mathbf{v}), \tag{40}$$

$$\rho_0 \frac{\partial}{\partial t}\mathbf{v} = \left(\frac{1}{\mu_0}\nabla \times \mathbf{B}_1 - \mathbf{j}_{h1\perp}'\right) \times \mathbf{B}_0 + \left(\frac{1}{\mu_0}\nabla \times \mathbf{B}_0 - \mathbf{j}_{h0\perp}'\right) \times \mathbf{B}_1 - \nabla P_1, \tag{41}$$

$$\frac{\partial \mathbf{B}_1}{\partial t} = -\nabla \times \mathbf{E}, \tag{42}$$

$$\frac{\partial P_1}{\partial t} = -\nabla \cdot (P_0\mathbf{v}) - (\gamma - 1)P_0\nabla \cdot \mathbf{v}, \tag{43}$$

$$\mathbf{j}_{h0\perp}' = \frac{1}{B_0}(P_{h\parallel 0}\nabla \times \mathbf{b}_0 - P_{h\perp 0}\nabla \ln B_0 \times \mathbf{b}_0) - \nabla \times \left(\frac{P_{h\perp 0}}{B_0}\mathbf{b}_0\right), \tag{44}$$

$$\begin{aligned}\mathbf{j}_{h1\perp}' = \frac{1}{B_0}(&P_{h\parallel 1}\nabla \times \mathbf{b}_0 + P_{h\parallel 0}\nabla \times \mathbf{b}_1 - P_{h\perp 1}\nabla \ln B_0 \times \mathbf{b}_0 \\ &- P_{h\perp 0}\nabla \ln B_0 \times \mathbf{b}_1) - \nabla \times \left(\frac{P_{h\perp 1}}{B_0}\mathbf{b}_0\right) - \nabla \times \left(\frac{P_{h\perp 0}}{B_0}\mathbf{b}_1\right),\end{aligned} \tag{45}$$

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B}_0. \tag{46}$$

For the calculation of $\mathbf{j}_{h1\perp}'$, terms with B_1 are neglected, since TAE modes are incompressible.

The initial condition is an MHD equilibrium, where the total plasma beta is 4% at the magnetic axis and its volume-average $\langle \beta_\alpha \rangle$ is 0.88%. Density is uniform. The cylindrical coordinate system (R, φ, z) is used. The simulation domain is $2a \leq R \leq 4a$, $-a \leq z \leq a$, where a is the minor radius. The magnetic axis locates at $R = R_0 \equiv 3.20a$, $z = 0$. Time is normalized with the Alfvén frequency ω_A which is defined at the magnetic axis through the Alfvén velocity and the major radius.

The initial energetic alpha particle distribution is the slowing-down distribution which is isotropic in the velocity space. The number density of alpha particles is in proportion to the MHD pressure. The sum of the alpha particle pressure and the MHD pressure is set equal to the equilibrium pressure. Here the alpha particle pressure is half the equilibrium pressure. The minimum and maximum energies of the slowing-down distribution are 0.05 and $1.8 m_\alpha v_A^2$, respectively. The Larmor frequency of the alpha particle is 102 times larger than the Alfvén frequency.

A fourth-order finite difference algorithm is used for the MHD equations. The number of grid points are 65×65 in the poloidal plane. Six runs with various number of particles are carried out. The numbers of used particles are 1×10^3, 4×10^3, 1.6×10^4, 6.4×10^4, 2.6×10^5, and 1.05×10^6, respectively.

Stability of $n = 2$ modes is investigated. The equilibrium is initially perturbed with an $n = 2$ magnetic field. An $n = 2$ TAE mode is destabilized in the simulation, and the radial profiles of the dominant poloidal harmonics of the electrostatic potential are shown in Fig. 1. This $n = 2$ TAE mode consists mainly of ($m = 2$, $n = 2$) and ($m = 3$, $n = 2$) harmonics. Time evolution of the real part of the ($m = 2$, $n = 2$) harmonic of δB_r at $r = 0.25a$ in the $N = 1.6 \times 10^4$ run is shown in Fig. 2. The real frequency is $0.33\omega_A$. Time evolutions of its amplitude in runs with $N = 1 \times 10^3$, $N = 4 \times 10^3$, and $N = 2.6 \times 10^5$ are shown in Fig. 3. An oscillation in the amplitude appears for $N = 1 \times 10^3$. This oscillation is a manifestation of destabilization of an additional $n = 2$ TAE mode which consists mainly of ($m = 3$, $n = 2$) and ($m = 4$, $n = 2$) harmonics and peaks around the $q = 7/4$ magnetic surface. The oscillation in amplitude is a beat between two TAE modes. The second $n = 2$ TAE mode is hidden in the other five runs, since its growth rate is much smaller than that of the first mode. For this point $N = 1 \times 10^3$ is not sufficient for study on $n = 2$ TAE modes.

The number of used particles and the linear growth rates are plotted in Fig. 4. The linear growth rate converges for large numbers of used particles. Comparing to the run with

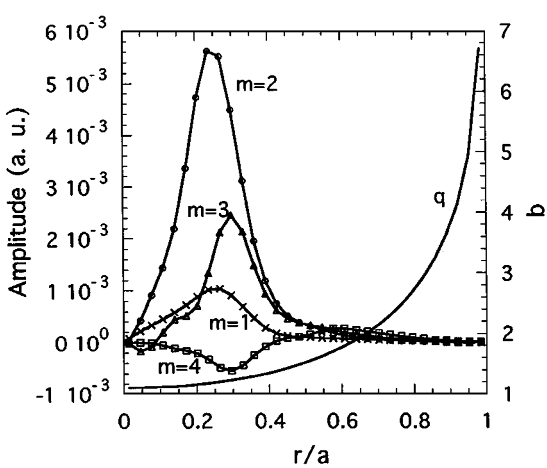

FIG. 1. Radial profiles of the dominant poloidal harmonics of the electrostatic potential of an $n = 2$ TAE mode and the q-profile.

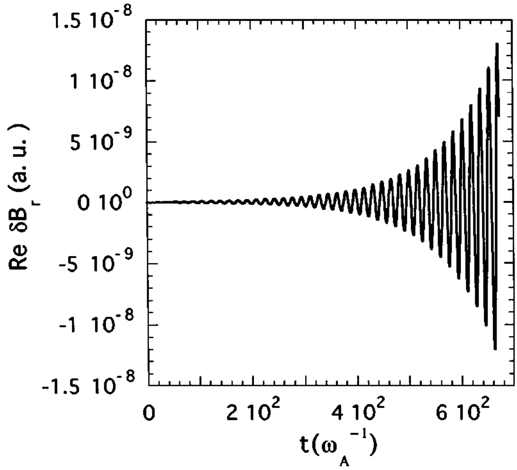

FIG. 2. Time evolution of real part of the $(m = 2, n = 2)$ harmonic of δB_r at $r = 0.25a$ in the $N = 1.6 \times 10^4$ run.The derived real frequency is $0.33\omega_A$.

$N = 1.05 \times 10^6$, deviations in growth rate of the runs with N larger than 4×10^3 are less than 10%. Therefore, we conclude that we can investigate the linear stability of an $n = 2$ TAE mode with 4×10^3 particles if we tolerate 10% error in growth rate.

4. SUMMARY AND DISCUSSION

A particle algorithm for linear kinetic analysis in tokamak plasmas is developed. Linear stability of a tokamak plasma is analyzed as an initial value problem. Particles are used to sample plasma elements along equilibrium characteristics in four-dimensional phase space $(R, z, v_\parallel, \mu)$. Each particle is accompanied with a weight which is a function of the toroidal angle. Destabilization of an $n = 2$ TAE mode is investigated. The linear growth

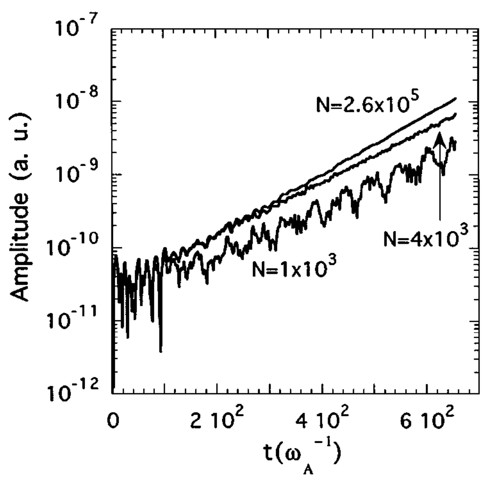

FIG. 3. Time evolutions of amplitude of the $(m = 2, n = 2)$ harmonic of δB_r at $r = 0.25a$ in runs with $N = 1 \times 10^3$, $N = 4 \times 10^3$, and $N = 2.6 \times 10^5$.

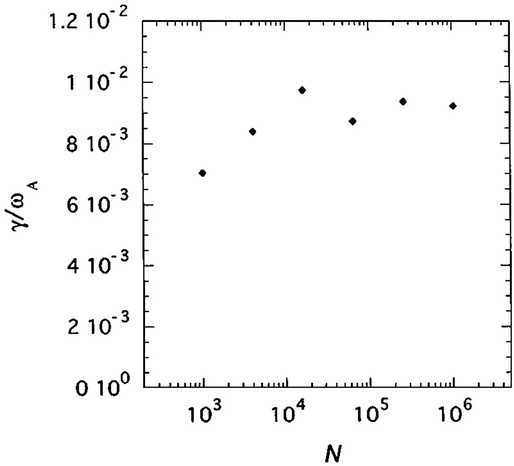

FIG. 4. Plotted is the linear growth rate for number of used particles.

rate converges for large numbers of used particles. We can investigate the linear stability of an $n = 2$ TAE mode with 4×10^3 particles if we tolerate 10% error in growth rate.

The small growth rate of TAE mode against its real frequency allows one to adopt a perturbative approach with preliminary MHD analysis. In this paper, however, we investigated the stability of the TAE mode without any preliminary analysis. With the new algorithm developed in this paper any preliminary analysis is not necessary. Furthermore, this nonperturbative approach enables us to investigate other kinetic issues in tokamak plasmas. For instance, analysis of energetic particle effects on the internal kink mode can be straightforward. The ion-temperature-gradient mode also can be analyzed if we apply the new algorithm to bulk ions and couple the ion density with the gyrokinetic Poisson equation [8].

ACKNOWLEDGMENTS

The authors sincerely thank all members of the Complexity Simulation Group for useful discussions. Numerical computations were performed at the Advanced Computing System for Complexity Simulation of the National Institute for Fusion Science. This work is partially supported by Grants-in-Aid of the Ministry of Education, Science, Culture, and Sports (No. 07832024 and No. 07858057).

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